Name:__________________________

SM3 2.1: Complex Factoring Techniques

Vocabulary: complex number

We're going to build onto some of the factoring techniques you learned during Secondary Math 2

$a^2 - b^2 = (a + b)(a - b)$

Difference of Squares: If two terms are both perfect squares and there is a – sign between them, you can write their factorization as the product of the roots of the terms being added and subtracted.

Review: Factor $x^2 - 9$

By now, you've realized that while the square root of some numbers is not an integer, the square root still exists. So we're going to relax the notion that both terms must be perfect squares.

Example: Factor $x^2 - 5$

$(x + \sqrt{5})(x - \sqrt{5})$ — $\sqrt{5}$ is the square root of 5.

Example: Factor $x^2 - 12$

$(x + \sqrt{12})(x - \sqrt{12})$ — $\sqrt{12}$ is the square root of 12.

$(x + 2\sqrt{3})(x - 2\sqrt{3})$ — $\sqrt{12}$ simplifies to $2\sqrt{3}$.

You've also seen that negative numbers have imaginary square roots, so we're going to remove the requirement of having the terms separated by a – sign.

Example: Factor $x^2 + 4$

$x^2 - (-4)$ — $+4$ can be written as $-(-4)$.

$(x + 2i)(x - 2i)$ — $2i$ is the square root of -4.

Example: Factor $x^2 + 7$

$x^2 - (-7)$ — $+7$ can be written as $-(-7)$.

$(x + i\sqrt{7})(x - i\sqrt{7})$ — $i\sqrt{7}$ is the square root of -7.

Practice: Factor each quadratic expression.

$x^2 - 11$ $\qquad$ $x^2 + 100$ $\qquad$ $x^2 + 13$ $\qquad$ $x^2 + 45$

$ax^2 + bx + c = (px + q)(sx + t)$

Factoring trinomials: Finding the factors of ac that sum to b and splitting bx in order to set up a workable grouping problem led to the factorization of any quadratic.

Review: Factor $2x^2 - 5x - 3$

Unfortunately, on occasion, we ran into quadratic trinomials that wouldn't be factored using the above technique. We know that all quadratics have two complex roots, and that each complex root is associated with a linear factor. We'll find the complex roots and then "unsolve" them into factors using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example: Factor $x^2 + 2x + 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

There are no factors of 5 that sum to 2, so we use the quadratic formula to find the roots of the quadratic.

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(5)}}{2(1)}$$

Substitution

$$x = \frac{-2 \pm \sqrt{4 - 20}}{2}$$

Multiplication

$$x = \frac{-2 \pm \sqrt{-16}}{2} = \frac{-2 \pm 4\mathrm{i}}{2}$$

Simplification

$$x = -1 \pm 2\mathrm{i}$$

We've got the roots of the quadratic. Let's write them both explicitly.

$$x = -1 + 2\mathrm{i} \qquad x = -1 - 2\mathrm{i}$$

$$x + 1 - 2i = 0 \qquad x + 1 + 2i = 0$$

Now let's move the terms in both equations to the left hand side to "unsolve" them into being factors.

$$(x + 1 - 2i)(x + 1 + 2i)$$

The left side of the equation contains the complex factors of the original polynomial.

It's safe to say that no student (or teacher for that matter) is going to come up with this factoring mentally. While this technique will work for all quadratics, the factoring skills you gained in your previous course are still faster; this technique should only be used when none of the faster methods work.

An important consideration is that there are infinitely many quadratics that contain the same two roots. For example, $x^2 + x + 1$ and $2x^2 + 2x + 2$ have the same roots because one is a multiple of the other. Using the quadratic formula to find the roots of these two polynomials will result in the same factors. You'll need to make certain that your factorization builds the right polynomial when distributed and this means selecting a lead coefficient factor of a when when $a \neq 1$.

Example: Factor $4x^2 + 2x + 3$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	There are no factors of 5 that sum to 2, so we use the quadratic formula to find the roots of the quadratic.
$x = \dfrac{-2 \pm \sqrt{2^2 - 4(4)(3)}}{2(4)}$	Substitution
$x = \dfrac{-2 \pm \sqrt{4 - 48}}{8}$	Multiplication
$x = \dfrac{-2 \pm \sqrt{-44}}{8} = \dfrac{-2 \pm 2\mathrm{i}\sqrt{11}}{8}$	Simplification
$x = -\dfrac{1}{4} \pm \dfrac{\mathrm{i}\sqrt{11}}{4}$	We've got the roots of the quadratic. Let's write them both explicitly.
$x = -\dfrac{1}{4} + \dfrac{\mathrm{i}\sqrt{11}}{4} \qquad x = -\dfrac{1}{4} - \dfrac{\mathrm{i}\sqrt{11}}{4}$	Now let's move the terms in both equations to the left hand side to "unsolve" them into being factors.
$x + \dfrac{1}{4} - \dfrac{\mathrm{i}\sqrt{11}}{4} = 0 \qquad x + \dfrac{1}{4} + \dfrac{\mathrm{i}\sqrt{11}}{4} = 0$	The left side of the equation contains the complex factors of the original polynomial.
$\left(x + \dfrac{1}{4} - \dfrac{\mathrm{i}\sqrt{11}}{4}\right)\left(x + \dfrac{1}{4} + \dfrac{\mathrm{i}\sqrt{11}}{4}\right)$	

But wait, we can quickly see that this factorization would produce a lead term of x^2. But our lead term is supposed to be $4x^2$. We didn't make an error; the quadratic formula is giving us the roots of our polynomial, but can't determine that $a \neq 1$. Since our polynomial has $a = 4$, we're going to have to put a 4 out front.

$$4\left(x + \frac{1}{4} - \frac{\mathrm{i}\sqrt{11}}{4}\right)\left(x + \frac{1}{4} + \frac{\mathrm{i}\sqrt{11}}{4}\right)$$

<u>HW2.1</u>

Factor each quadratic expression completely over the set of complex numbers.

1) $12m^2 + 12$

2) $7x^2 + 2$

3) $-4x^2 - 8$

4) $4x^2 + 7$

5) $11x^2 + 6$

6) $2v^2 + 7$

7) $4x^2 + 4x + 5$

8) $4x^2 + 8x + 6$

9) $9n^2 - 5n + 12$

10) $8p^2 + 5p + 2$

11) $3n^2 - 7n + 12$

12) $10a^2 + 3$

13) $4a^2 + 12a + 10$

14) $9n^2 - 12n + 10$

15) $12v^2 - 3v + 2$

16) $2n^2 + 6$

17) $12a^2 + 7$

18) $9x^2 + 12x + 6$

19) $4x^2 - 7x + 4$

20) $6r^2 - 4r + 11$